Defuzzification in Medical Diagnosis*

*in Abe JM, Silva Filho JI (Editors). Advances in logic, artificial intelligence and robotics. Amsterdam: IOS Press, 2002, pages 202 – 207

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Abstract

In this work, we present a classification of patients through relational fuzzy signs/disease matrices. These relational fuzzy matrices were elaborated based on real cases and through two methods: max-min composition and Gödel implication. The results found from the different methods were compared and discussed in the medical point of view. The necessity of the defuzzification process in a medical environment is also discussed.

Keywords: Fuzzy Relations, max-min Composition, Gödel Implication, Medical diagnosis, Defuzzification.

1. Introduction

Fuzzy systems concerning medical diagnosis often seek the allocation of patients to specific nosologic categories through some sort of defuzzification rule. This follows the rationale of medical practice under which for a patient presenting with a given set of clinical signs and symptoms a conclusive diagnosis should be produced.

Such conclusion, nonetheless, will always hinge on premises of previous composite relational equations, which analyzing records of past patients concerning symptoms & signs and diagnoses, establish how they are related. Depending on the model used in this analysis, different sort of information will be available for the definition of a defuzzification procedure. In the present paper, the yields of two different methods are studied.

2. Methods

A data set of 153 children recording their clinical signs and diagnoses was randomly separated into an analysis sample (75% of cases) and a validation sample (25% of cases), the former been used to derive a matrix of relations between diagnoses and signs, and the latter been used to assess performance in patient allocation to a diagnosis category according with the aforementioned matrix. Diagnoses were pneumonia (d_1), diseased but not pneumonia (d_2), and healthy (d_3), crisply assessed as either present or absent. Clinical signs were chest X-ray (s_1), dyspnoea (s_2), chest auscultation (s_3), cardiac rate (s_4), body temperature (s_5), toxaemia (s_6), respiratory rate (s_7), which were originally assessed through qualitative scales, whose values were normalised to the unit to express different grades of membership as a linear relation between absence and maximum presentation.

The relational models compared were a max-min (sup-min) composition and Gödel's implication [1,2,3]. Max-min relation is defined as:

Max-min relation: Given S and T, two binary relations of U × V and V × W (*e.g. signs* × *patient and patient* × *diagnosis*), the composition sup-min (*e.g. signs* × *diagnoses*) is a fuzzy relation in U × W of the type

$$(S*T)(x,z) = \sup_{y\in V}[\min(S(x,y),T(y,z))] \qquad (1)$$

Gödel´s implication: Gödel's implication treats Cartesian products in a way that preserves the truth table. The relational equation that incorporates Gödel's implication is given by

$$(S *_g T)(x,z) = \inf_{y\in V}[g(S(x,y),T(y,z))] \qquad (2)$$

and g is defined as a function in [0,1] × [0,1], such that:

$$g : [0,1] \times [0,1] \rightarrow [0,1]$$

where

$$\begin{aligned} g(a,b) &= a \Rightarrow b \\ &= \sup\{x \in [0,1] : \min(a,x) \le b\} \\ &= \begin{cases} 1 & if \quad a \le b \\ b & if \quad a > b. \end{cases} \end{aligned}$$

To allocate patients from the validation sample to the diagnosis categories, for each patient (P_n) allocation for diagnoses (d_m) was drawn from the composition of his vector of clinical signs (s_i) with the relational matrix of each model according with the function

$$R(P_n)(d_m) = \sup_{1\le i\le 7}[\min[R(d_m,s_i),P_n(s_i)]] \qquad (3)$$

The defuzzification rule was defined as the maximum points of the resulting membership functions [1], in other words, one patient should be allocated to the diagnosis to which he had highest membership. As the relation between signs and healthy could achieve some degree of membership, a patient should be allocated to the healthy category if his membership to both pneumonia and other disease were null.

To assess performance of defuzzification under each model, the overall agreement between results and the known classification of patients in the validation sample was calculated.

2. Results

Derived from the analysis sample, the matrixes of relations between clinical signs and diagnoses under each model were:

Table 1 - Relationship matrixes for the two models studied.

max-min

	d_1	d_2	d_3
s_1	0.43	0	0
s_2	1	0.25	0
s_3	0.67	0	0
s_4	1	1	0.5
s_5	0.67	1	0
s_6	0.75	0.5	0
s_7	1	0.75	0

Gödel

	d_1	d_2	d_3
s_1	1	0	0
s_2	0	0	0
s_3	1	0	0
s_4	0	0	0
s_5	0	0	0
s_6	0	0	0
s_7	0	0	0

Max-min composition better elicits relations of intersection, or equivalence, or interspersing, and thus even healthy subjects (d_3) can present with some degree of a clinical sign, which resulted in cardiac rate (s_4) having some relation with the healthy diagnosis (actually, 6 children had mild tachycardia, 4 of which had weights below the 25º percentile, thus probably not being perfectly healthy but having malnutrition and the not rarely accompanying anemia, which could account for the tachycardia). Gödel exams implication, thus causal relationship, and in the present study, as a corollary of the diagnoses being crisply assessed, this resulted in selecting signs which were pathognomonic to pneumonia (s_1 and s_3, chest X-ray and chest auscultation).

Applying equation (3) for each model and patient, a vector of values representing membership of each patient to diagnoses 1 to 3, namely pneumonia, diseased not pneumonia, and healthy, was accomplished. This had the form of what is presented below for a sample of patients:

Table 2 - Patient membership to diagnosis category according to each model.

max-min

Patient ID	pneumonia	other disease	healthy
5	0.75	0.5	0.5
10	0.25	0.25	0.25
23	0.75	0.5	0
31	0.75	0.75	0.5
37	0.75	0.75	0.5
38	0.33	0.25	0.25
etc.			

Gödel

Patient ID	pneumonia	other disease	healthy
5	0.43	0	0
10	0.14	0	0
23	0.33	0	0
31	0.67	0	0
37	0.67	0	0
38	0.33	0	0
etc.			

Applying the defined defuzzification rule, in max-min model there were situations when allocation of patients was missed due to ties in membership functions, e.g. patient nº 10 under. An alternative was to allow multiple allocation, so that a patient could be simultaneously allocated to more than one category. Agreement between allocation of patients through the two models information and the patient real status in the validation sample is shown in tables 3 to 5:

Table 3 - Patient allocation according with max-min relation matrix taken as a single response variable.

		patient real status			
		pneumonia	other disease	healthy	Total
max-min	pneumonia	3			3
	other disease				
	healthy		4	11	15
	Total	3	4	11	18
Overall agreement: 77.8% Valid cases: 18 Missing cases: 20					

Table 4 - Patient allocation according with max-min relation matrix taken as a multi response variable.

		patient real status			
		pneumonia	other disease	healthy	Total
max-min	pneumonia	10	9	4	23
	other disease	7	9	4	20
	healthy		4	11	15
	Total	10	13	15	38
Overall agreement: 78.3% Valid cases: 38 Missing cases: 0					

Table 5 - Patient allocation according with Gödel's relation matrix.

		patient real status			
		pneumonia	other disease	Healthy	Total
Gödel	pneumonia	10			10
	other disease				
	healthy		13	15	28
	Total	10	13	15	38
Overall agreement: 65.8% Valid cases: 38					

3. Comments and Conclusions

Where is the knowledge we have lost in information, where is the wisdom we have lost in knowledge.

T.S. Elliot

A defuzzification procedure may be inescapable in fuzzy control systems as for a given fuzzy output a specific action ought to be taken in an automated engineering process. Nonetheless, when dealing with medical diagnosis, one wonders whether this is indeed required. Would not such procedure jeopardize the wealth of knowledge produced by fuzzy set theory and betray wisdom, or, in other words, would not defuzzification narrow the view one can get of a given subject instead of enlarging it?

In the present exercise, one could perhaps conclude that comparing the two models tested, both could be considered with a fair performance on predicting a patient's diagnosis since both have more correct than wrong hits. Max-min approach could be reckoned to some extent superior to Gödel's and the intermediate situation of being sick, but not bearing pneumonia could be deemed as poorly assessed by the set of clinical signs taken into analysis – indeed, under both models, classification of patients with other disease results rather challenging. Nonetheless, if defuzzification is left apart and results are offered as grades of membership to the different nosologic categories (table 2) much more information is provided to decision-making. Taking, for instance, the awkward case of patient nº 10, instead of allocating him to any given diagnosis by choosing any defuzzification rule, one would be much better off with the information that max-min suggests that he is equally weakly associated with the three diagnoses and that Gödel adds to this that he has some degree of a pathognomonic sign of pneumonia. Decision as whether his diagnosis is this or that should better be ultimated by medical judgement, which would use this as complementary information in its decision taking process.

Physicians endeavor to attain a diagnosis in order to trigger a set of corresponding actions concerned with re-establishing health, and these actions are the real final goal of medical care. Sadegh-Zadeh [4], giving emphasis to the fact that misdiagnosis might be as frequent as 40% in all patients seen, suggests that fuzzy medical systems should focus on actions to be taken instead of on diagnoses, which by their turn should be only one additional piece of information in the decision taking process. Considering this, it seems that a fuzzy medical system would better provide subsidiary information than experimenting with superseding the physician role.

Nguyen and Walker [5] , discussing defuzzification state that "*Standard control theory is efficient when precise mathematical models are available. The emergence of artificial intelligence technologies has suggested the additional use of manual control expertise in more complex problems*." Medical systems are undoubtedly the case and last decision should be left to manual control expertise.

Medical sciences are concerned with causal relationships and, according with Susser [6], different frames of reference provide different views of such relations. Knowledge is completed through an exercise of bringing together different views and the present study was exemplary in this sense. With the information provided in table 1 one can learn, for instance, that chest X-ray and chest auscultation, in spite of being pathognomonic to pneumonia (Gödel's model), are not the clinical signs with the largest intersection with this diagnosis: dyspnoea, tachycardia and tachypnea (s_2, s_4, s_7) have much higher degrees of membership (max-min model). This is in agreement with the current medical belief that inspection of patients has precedence over radiological or laboratory investigation and reassures that, indeed, if either chest X-ray or auscultation is positive a diagnosis of pneumonia is very likely, but other clinical signs should not go unattended as they are as much or even more informative.

Likewise, finding out that the healthy status had some degree of membership to cardiac rate allowed the identification of a few cases that had been considered healthy, when they would have been better classified as bearers of some disease, either malnutrition or anemia. Further exploring this table, one can learn how each signal relate to each diagnosis and thus for which situation it is more important – for instance, toxaemia (s_6), even though not pathognomonic to pneumonia, is more important to this diagnosis than to the diagnosis of other disease.

Proceeding to table 2, where relations of each patient to each diagnosis are shown, one can realize that any patient is better described by his vector of relations with diagnoses than by a single allocation to a specific diagnosis. Table 4 provides evidence of this since it shows that if multiple allocation is allowed, the level of agreement is enhanced and no cases are left out of classification.

In conclusion, the results of this study were suggestive that fuzzy diagnostic systems should perhaps rule out defuzzification in favor of providing information about relations and leaving decision to medical judgement. Shirking defuzzification one should be avoiding the loss of wisdom that the poet warned against. Giving emphasis to relations one should be achieving real knowledge since according with Poincaré [7] "*outside relations there is no reality knowable*".

Acknowledgement: This study was supported by FAPESP, the São Paulo State Research

Supporting Agency, grant nº 01/04905-0

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